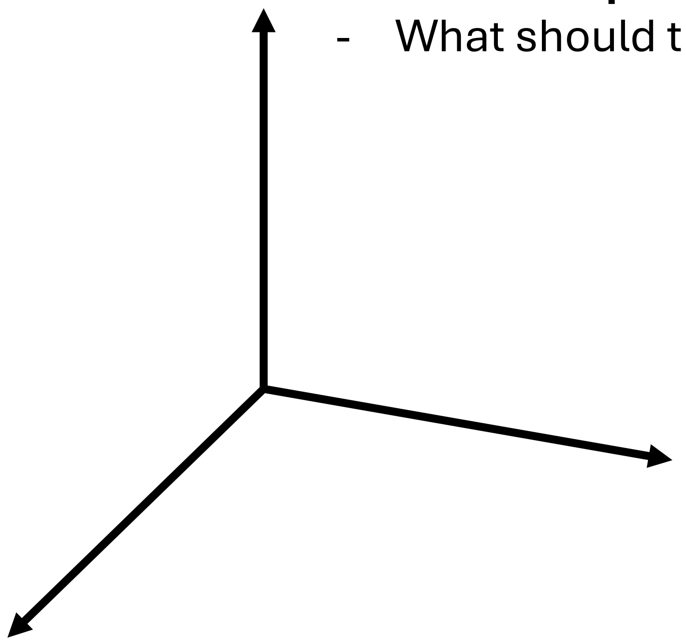
Machine Programming

Lecture 4 – Functional Specifications for Synthesis

Ziyang Li

Dimensions in Program Synthesis

Behavioral Specification
- What should the program do?



Synthesis Strategy
- How do we find such a program?

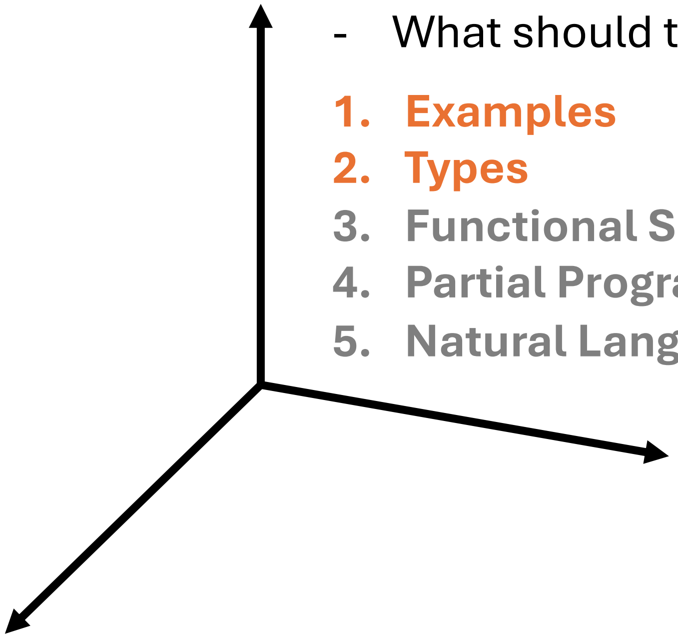
Structural Specification
- What is the space of the programs?

The Course So Far

Behavioral Specification

- What should the program do?

1. **Examples**
2. **Types**
3. **Functional Specifications**
4. **Partial Programs**
5. **Natural Language**



Synthesis Strategy

- How do we find such a program?

Enumeration

- Enumerating all programs with a grammar
- Bottom-up vs top-down

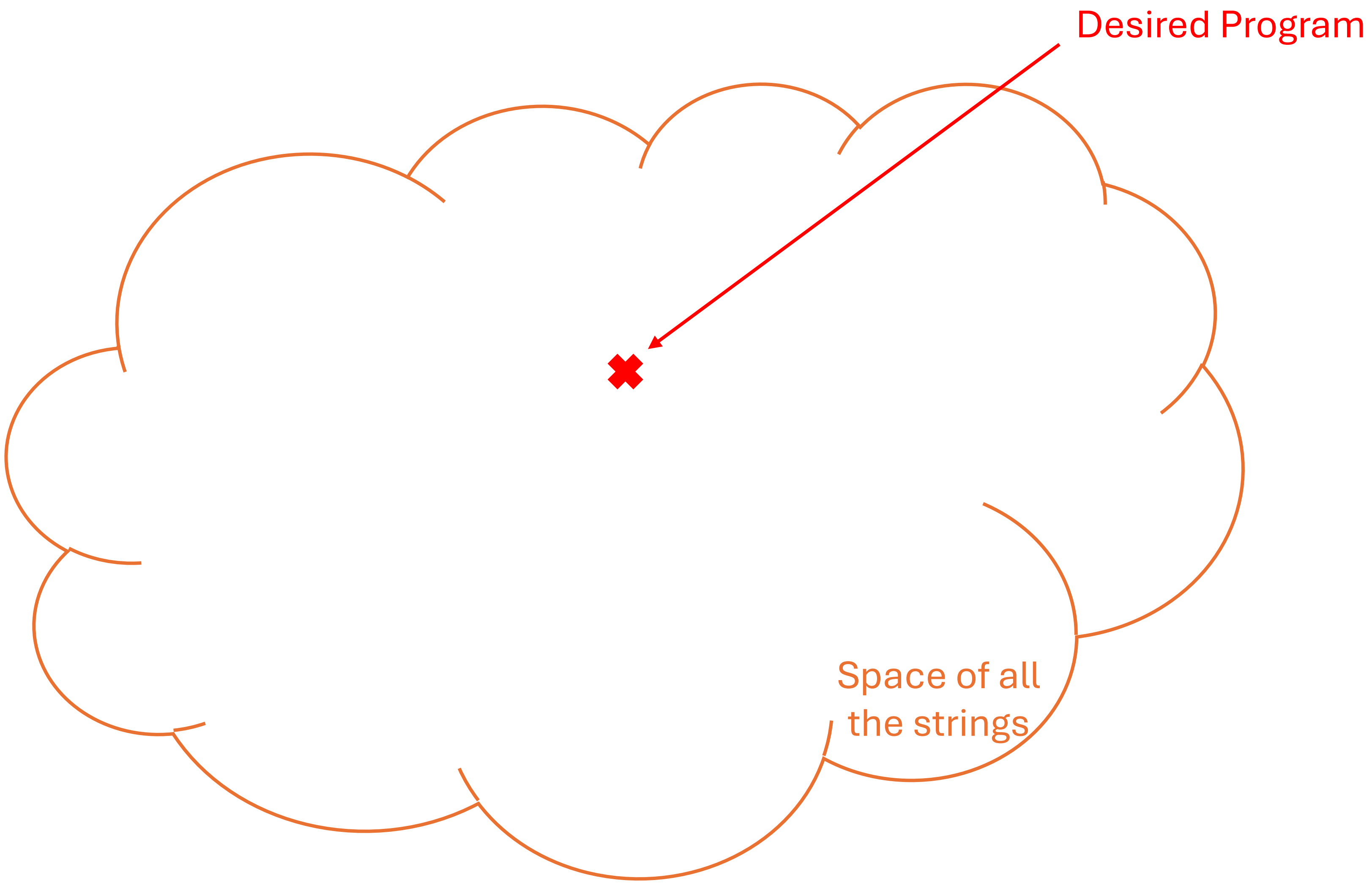
Structural Specification

- What is the space of the programs?

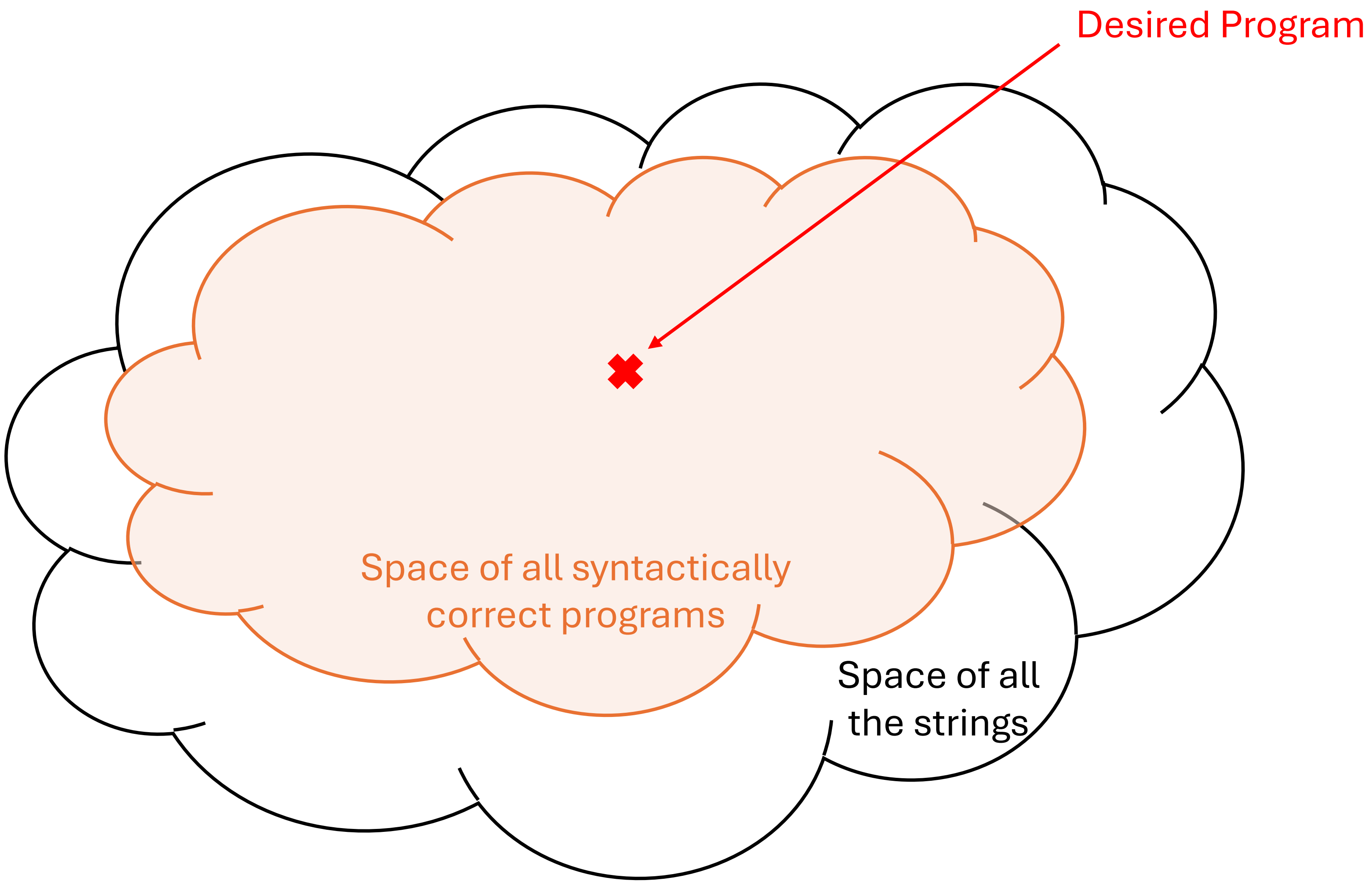
Context-Free / Regular Tree Grammar

Expr e ::= c | e + e | e * e

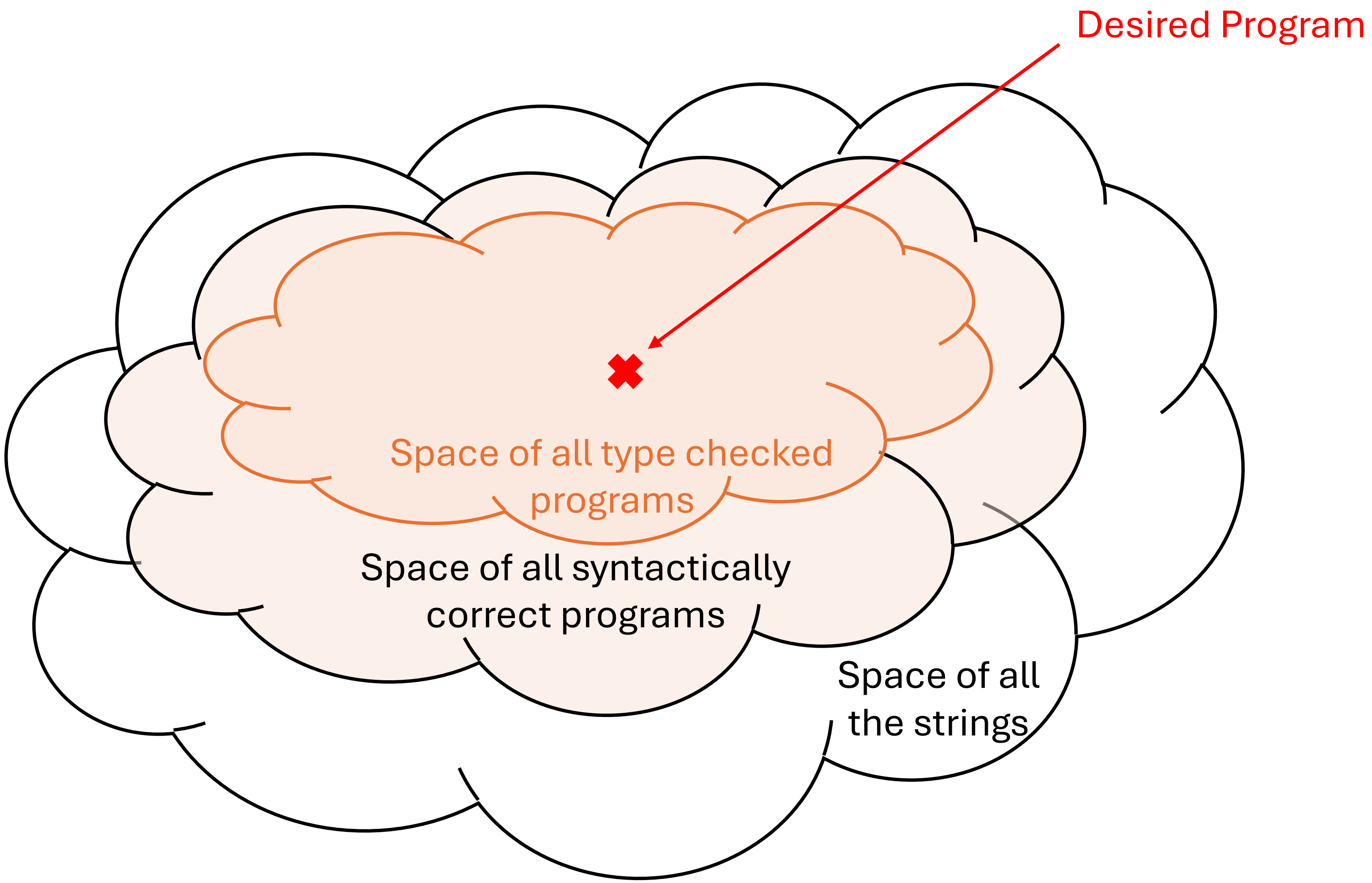
High Level Picture



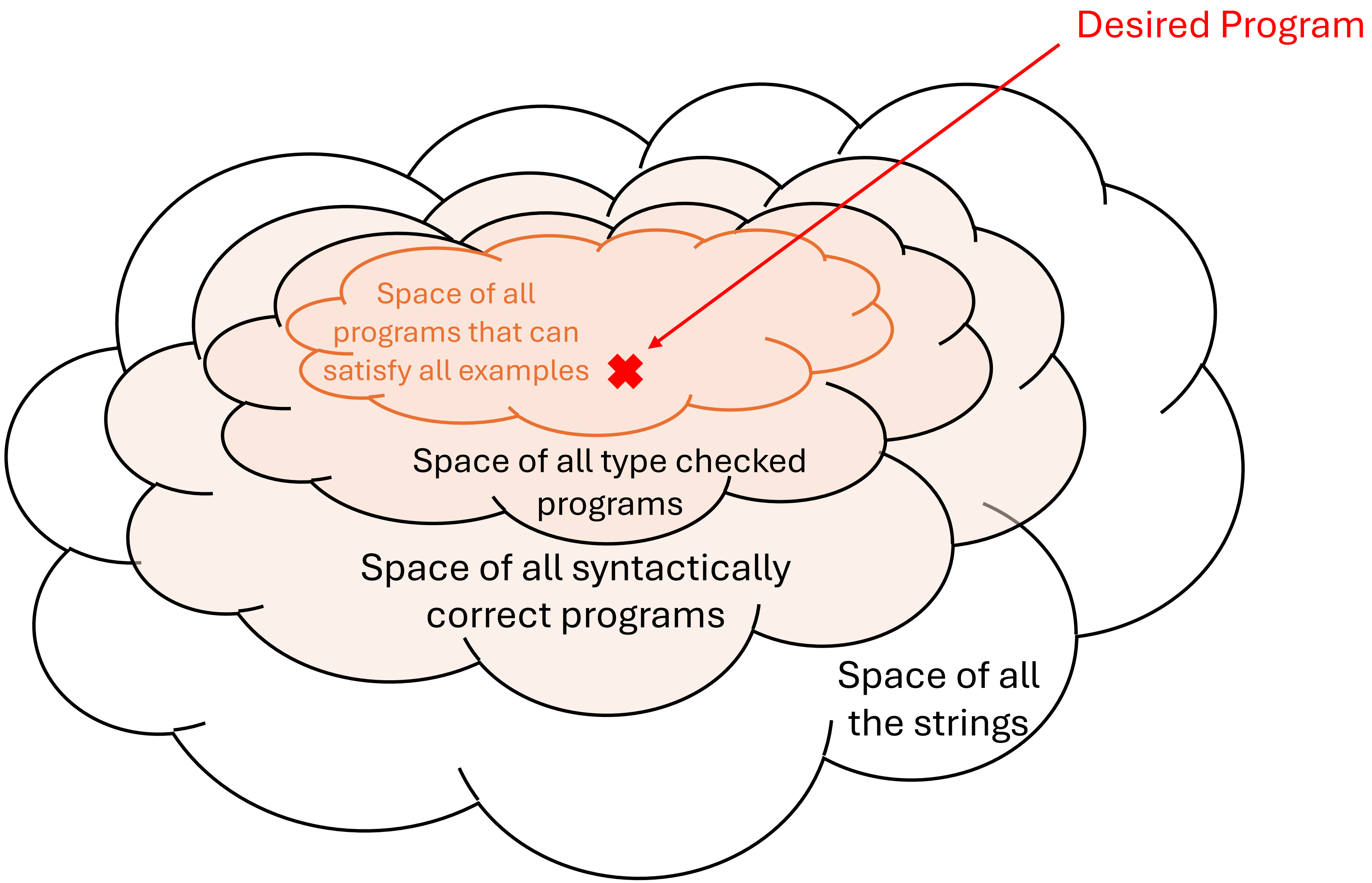
High Level Picture



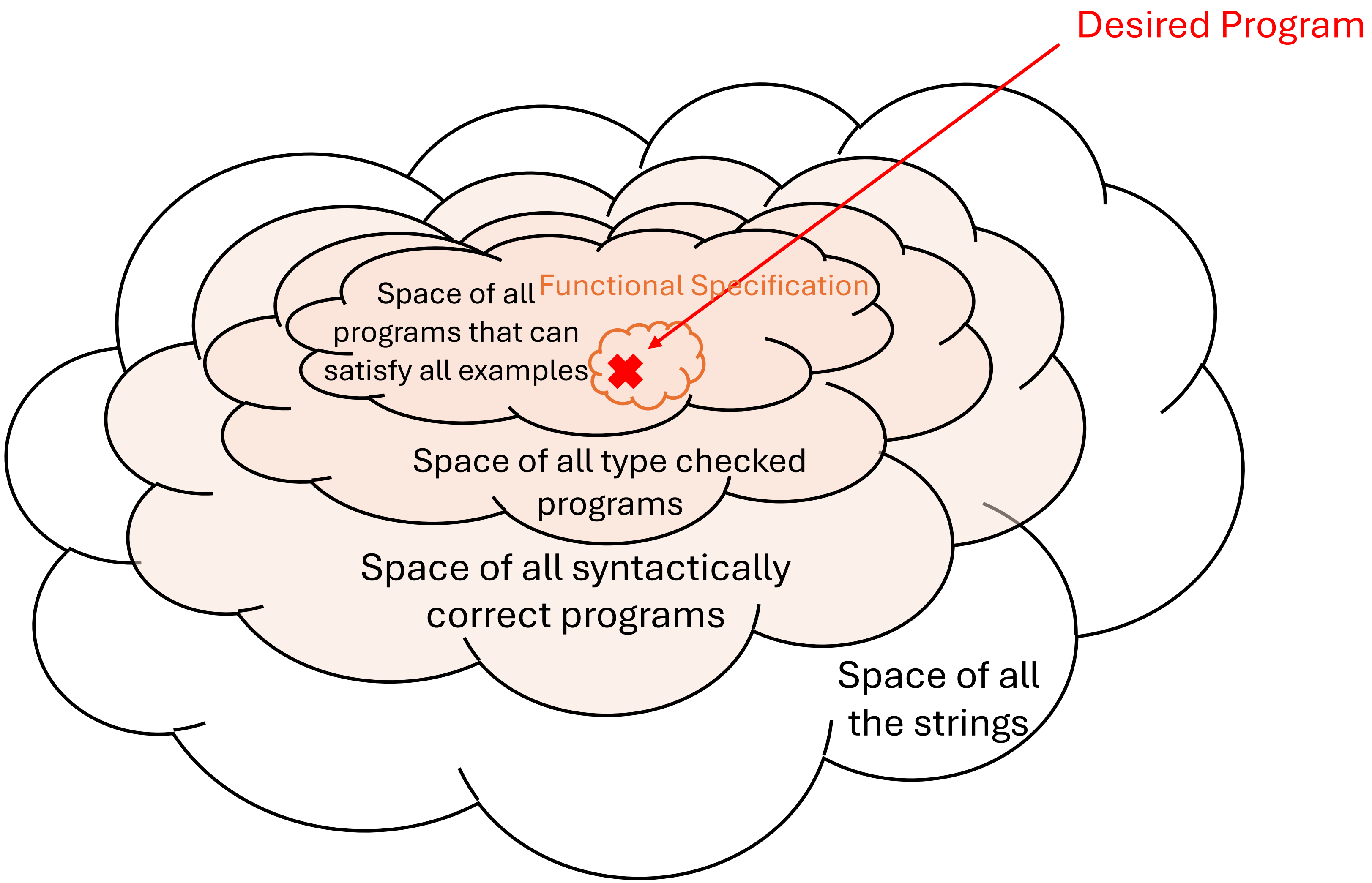
High Level Picture



High Level Picture



High Level Picture



Today

Behavioral Specification

- What should the program do?

1. Examples
2. Types
3. Functional Specifications
4. Parametric Programs
5. Natural Language

Synthesis Strategy

- How do we find such a program?

Structural Specification

- What is the space of the programs?

Is Examples Enough?

```
F([3, 2, 1]) = [1, 2, 3]
F([2, 1]) = [1, 2]
F([]) = []
```

Is Examples Enough?

```
F([3, 2, 1]) = [1, 2, 3]
F([2, 1]) = [1, 2]
F([]) = []
```

What is F?

Is Examples Enough?

```
reverse([3, 2, 1]) = [1, 2, 3]
reverse([2, 1]) = [1, 2]
reverse([]) = []
```

What is F?

Is Examples Enough?

```
sort([3, 2, 1]) = [1, 2, 3]
   sort([2, 1]) = [1, 2]
       sort([]) = []
```

What is F?

Is Examples Enough?

```
reverse([3, 2, 1]) = [1, 2, 3]
   reverse([2, 1]) = [1, 2]
      reverse([]) = []

   sort([3, 2, 1]) = [1, 2, 3]
      sort([2, 1]) = [1, 2]
         sort([]) = []
```

What is F?

Is Type Enough?

```
reverse : List[int] -> List[int]
reverse([3, 2, 1]) = [1, 2, 3]
reverse([2, 1]) = [1, 2]
reverse([]) = []

sort : List[int] -> List[int]
sort([3, 2, 1]) = [1, 2, 3]
sort([2, 1]) = [1, 2]
sort([]) = []
```

Functional Specifications

$$\forall x, y \in \text{List[Int]}, \boldsymbol{reverse}(x) = y \Rightarrow$$
$$len(x) = len(y) \land \forall i \in \{1 \ldots len(x)\}, x_i = y_{len(y)+1-i}$$

```
reverse([3, 2, 1]) = [1, 2, 3]
reverse([2, 1]) = [1, 2]
reverse([]) = []
```

$$\forall x, y \in \text{List[Int]}, \boldsymbol{sort}(x) = y \Rightarrow$$
$$len(x) = len(y) \land \forall i \in [0, len(x) - 1), y_{i+1} > y_i$$

```
sort([3, 2, 1]) = [1, 2, 3]
sort([2, 1]) = [1, 2]
sort([]) = []
```

Functional Specifications

$\forall x, y \in \text{List[Int]}, \boldsymbol{reverse}(x) = y \Rightarrow$

$$len(x) = len(y) \land \forall i \in \{1 \dots len(x)\}, x_i = y_{len(y)+1-i}$$

1. Input and output has the same length;
2. The i-th element in x is the same as $(n + 1 - i)$-th element in y

```
reverse([3, 2, 1]) = [1, 2, 3]
reverse([2, 1]) = [1, 2]
reverse([]) = []
```

$\forall x, y \in \text{List[Int]}, \boldsymbol{sort}(x) = y \Rightarrow$

$$len(x) = len(y) \land \forall i \in [0, len(x) - 1), y_{i+1} > y_i$$

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sort([3, 2, 1]) = [1, 2, 3]
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Functional Specifications

$\forall x, y \in \text{List[Int]}, \boldsymbol{reverse}(x) = y \Rightarrow$

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1. Input and output has the same length;
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```
reverse([3, 2, 1]) = [1, 2, 3]
reverse([2, 1]) = [1, 2]
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```

$\forall x, y \in \text{List[Int]}, \boldsymbol{sort}(x) = y \Rightarrow$

$$len(x) = len(y) \land \forall i \in [0, len(x) - 1), y_{i+1} > y_i$$

1. Input and output has the same length;
2. In result y, the i-th element is always less than or equal to the $(i + 1)$-th element

```
sort([3, 2, 1]) = [1, 2, 3]
sort([2, 1]) = [1, 2]
sort([]) = []
```

Functional Specifications

$$\forall x, y \in \text{List[Int]}, \boldsymbol{sort}(x) = y \Rightarrow$$
$$len(x) = len(y) \wedge \forall i \in [0, len(x) - 1), y_{i+1} > y_i$$

Pre-condition:

Post-condition:

Functional Specifications

Pre-condition:

$x \in \text{List[Int]}$ x is an integer list

$\forall x, y \in \text{List[Int]}, \boldsymbol{sort}(x) = y \Rightarrow$

$len(x) = len(y) \wedge \forall i \in [0, len(x) - 1), y_{i+1} > y_i$

Post-condition:

Functional Specifications

$$\forall x, y \in \text{List[Int]}, \boldsymbol{sort}(x) = y \Rightarrow$$
$$len(x) = len(y) \wedge \forall i \in [0, len(x) - 1), y_{i+1} > y_i$$

Pre-condition:

$x \in \text{List[Int]}$	x is an integer list

Post-condition:

$y \in \text{List[Int]}$	y is an integer list
$len(x) = len(y)$	y and x has the same length
$y_{i+1} \geq y_i$	Latter element in y always bigger than or equal to previous element

Form: Pre- and Post-Conditions

Premise: All valid inputs to a function MUST satisfy

Pre-condition:

$x \in \text{List[Int]}$	x is an integer list

Promise: all outputs WILL satisfy if the premise holds

$$\forall x, y \in \text{List[Int]}, \boldsymbol{sort}(x) = y \Rightarrow$$
$$len(x) = len(y) \wedge \forall i \in [0, len(x) - 1), y_{i+1} > y_i$$

Post-condition:

$y \in \text{List[Int]}$	y is an integer list
$len(x) = len(y)$	y and x has the same length
$y_{i+1} \geq y_i$	Latter element in y always bigger than or equal to previous element

Pre- and Post-Conditions in the Wild

Dafny

```
method Find(a: array<int>, key: int) returns (index: int)
  ensures 0 <= index ==> index < a.Length && a[index] == key
  ensures index < 0 ==> forall k :: 0 <= k < a.Length ==> a[k] != key
{
  index := 0;
  while index < a.Length
  {
    if a[index] == key { return; }
    index := index + 1;
  }
  index := -1;
}
```

Checked-C

```
int abs_val(int x)
  _Pre_ true
  _Post_ (retval >= 0)
{
  if (x < 0) {
    return -x;
  } else {
    return x;
  }
}
```

SV-Comp (C)

```
int main() {
  int i;
  int j=__VERIFIER_nondet_int();
  int n=__VERIFIER_nondet_int();
  assume_abort_if_not(n < 100000);
  int a[n];

  assume_abort_if_not(j>0 && j < 10000);
  for(i=1;i<n;i++) {
    int k=__VERIFIER_nondet_int();
    assume_abort_if_not(k>0 && k < 10000);
    a[i]=i+j+k;
  }

  for(i=1;i<n;i++)
    __VERIFIER_assert(a[i]>=(i+2));
  return 0;
}
```

Russol (Rust)

```
#[requires(self.len() > 0)]
fn peek(&self) -> &T {
  todo!()
}
```

Software Foundations / Hoare Logic (Coq)

```
{{ True }}
while X ≠ 0 do
  X := X - 1
end
{{ X = 0 }}
```

Pre- and Post-Condition Practice

```python
def insert(list: List[int], elem: int) -> List[int]:
```

```python
def sqrt(x: float) -> float:
```

```python
def matrix_mul(a: Tensor, b: Tensor) -> Tensor:
```

Completeness of Pre- and Post-Conditions

Pre-condition $x \in \text{List[Int]} \quad n = len(x)$

```
y = sort(x)
```

Is this post-condition complete?

Post-condition $y \in \text{List[Int]} \quad \forall i.\, 0 \leq i < n - 1 \Rightarrow y[i] \leq y[i+1]$

Completeness of Pre- and Post-Conditions

Input: [1, 2, 3, 4, 5]

Pre-condition $x \in \text{List[Int]}$ $n = len(x)$

```
y = sort(x)
```

Is this post-condition complete? ❌

Post-condition $y \in \text{List[Int]}$ $\forall i.\, 0 \le i < n - 1 \Rightarrow y[i] \le y[i+1]$

Output: [1, 2, 3, 4, 5, 6]

Completeness of Pre- and Post-Conditions

Pre-condition $x \in \text{List[Int]}$ $n = len(x)$

```
y = sort(x)
```

Attempt 2:
Is this post-condition complete?

Post-condition $y \in \text{List[Int]}$ $\forall i.\, 0 \leq i < n - 1 \Rightarrow y[i] \leq y[i+1]$

$len(y) = n$

Completeness of Pre- and Post-Conditions

Input: [1, 2, 3, 4, 5]

Pre-condition $x \in \text{List[Int]}$ $n = len(x)$

```
y = sort(x)
```

Attempt 2:
Is this post-condition complete? ❌

Post-condition $y \in \text{List[Int]}$ $\forall i.\, 0 \leq i < n - 1 \Rightarrow y[i] \leq y[i+1]$

$len(y) = n$

Output: [5, 5, 5, 5, 5]

Completeness of Pre- and Post-Conditions

Pre-condition $x \in \text{List}[\text{Int}]$ $n = len(x)$

```
y = sort(x)
```

Attempt 3:
Is this post-condition complete?

Post-condition $y \in \text{List}[\text{Int}]$ $\forall i.\, 0 \leq i < n - 1 \Rightarrow y[i] \leq y[i+1]$

$len(y) = n$

$\forall i.\, 0 \leq i < n \Rightarrow \exists j.\, x[i] = y[j]$

Completeness of Pre- and Post-Conditions

Pre-condition $x \in \text{List[Int]}$ $n = len(x)$

```
y = sort(x)
```

Attempt 3:
Is this post-condition complete?

Post-condition $y \in \text{List[Int]}$ $\forall i.\, 0 \leq i < n - 1 \Rightarrow y[i] \leq y[i + 1]$

$len(y) = n$

$\forall i.\, 0 \leq i < n \Rightarrow \exists j.\, x[i] = y[j]$

$\forall i.\, 0 \leq i < n \Rightarrow \exists j.\, y[i] = x[j]$

Completeness of Pre- and Post-Conditions

Input: [1, 2, 3, 4, 2]

Pre-condition $x \in \text{List[Int]}$ $n = len(x)$

```
y = sort(x)
```

Attempt 3:
Is this post-condition complete? ❌

Post-condition $y \in \text{List[Int]}$ $\forall i. 0 \leq i < n - 1 \Rightarrow y[i] \leq y[i + 1]$

Output: [1, 2, 3, 4, 4]

$len(y) = n$

$\forall i. 0 \leq i < n \Rightarrow \exists j. x[i] = y[j]$

$\forall i. 0 \leq i < n \Rightarrow \exists j. y[i] = x[j]$

Completeness of Pre- and Post-Conditions

Pre-condition $x \in \text{List[Int]}$ $n = len(x)$

```
y = sort(x)
```

Attempt 4:
Is this post-condition complete? ✅

Post-condition $y \in \text{List[Int]}$ $\forall i.\, 0 \leq i < n - 1 \Rightarrow y[i] \leq y[i+1]$

$len(y) = n$

$\exists p: \mathbb{Z}_n \rightarrow \mathbb{Z}_n, p \text{ is a permutation},$
$\forall i.\, 0 \leq i < n \Rightarrow y[i] = x[p(i)]$

Difficult to specify complete functional specification!

Leveraging Rust Types for Program Synthesis

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Rust type +
functional spec

```
[#requires …]
[#ensures …]
fn target(x: T1…) -> T

[#pure]
fn f(x: T1…) -> T
…
```

Contributions. In summary, this paper makes the following contributions:

- Synthetic Ownership Logic (SOL), a variant of Separation Logic that is targeted to program synthesis of well-typed Rust programs from type signatures and functional specifications.
- RusSOL, the first synthesizer for Rust code from functional correctness specifications. We built RusSOL by integrating SOL into SuSLik's general-purpose proof search framework.
- An extensive evaluation of RusSOL with regard to utility and performance. We show that it is capable of synthesizing a large number of non-trivial heap-manipulating Rust programs, in a matter of seconds, and that $\boxed{\text{required annotations are on average 27\% shorter than the code.}}$

Specification itself has a language

- First-order logic operators:
  - $\forall, \exists, =, \neq, \wedge, \vee, \neg, \Rightarrow, \ldots$
- Base types, commonly used types, and their predicates
  - Int, Bool, Set[T], List[T], …
  - $+, -, \times, \div, ::, \cup, \cap, \in, [.], \&\&, ||, !, len, \ldots$
- Logic systems to specify program behaviors
  - Memory (Separation Logic): $\{.\}, \mapsto, *, \ldots$
  - Temporal Behavior (Temporal Logic): $\mathrm{Globally}, \mathrm{Next}, \mathrm{Finally}, \ldots$
  - Mathematical Objects: Permutation, …

$\{x \mapsto a * y \mapsto b\}$ `void swap(loc x, loc y)` $\{x \mapsto b * y \mapsto a\}$

Structuring the Synthesis of Heap-Manipulating Programs, Polikarpova et. al., 2019

Specification itself has a language

Syntax + Semantics

- First-order logic operators:
  - $\forall, \exists, =, \neq, \wedge, \vee, \neg, \Rightarrow, \ldots$
- Base types, commonly used types, and their predicates
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- Logic systems to specify program behaviors
  - Memory (Separation Logic): $\{.\}, \mapsto, *, \ldots$
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???

Specification itself has a language

Syntax + Semantics

Syntax

- First-order logic operators:
  - $\forall, \exists, =, \neq, \wedge, \vee, \neg, \Rightarrow, \ldots$
- Base types, commonly used types, and their predicates
  - Int, Bool, Set[T], List[T], …
  - $+, -, \times, \div, ::, \cup, \cap, \in, [.], \&\&, ||, !, len, \ldots$
- Logic systems to specify program behaviors
  - Memory (Separation Logic): $\{.\}, \mapsto, *, \ldots$
  - Temporal Behavior (Temporal Logic): $\mathrm{Globally, Next, Finally}, \ldots$
  - Mathematical Objects: Permutation, …

Semantics

$$\{P\}\ c\ \{Q\}$$

Hoare Triple

$$[[c]] \vDash Q$$

Operational Alignment

Verification of Program with a Specification

{Pre-condition} Program {Post-condition}

$$\{P\}\ c\ \{Q\}$$

Verification of Program with a Specification

{Pre-condition} Program {Post-condition}

$$\{P\}\ c\ \{Q\}$$

$\{x \mapsto a * y \mapsto b\}$ `void swap(loc x, loc y)` $\{x \mapsto b * y \mapsto a\}$

```
{{ True }}
while X ≠ 0 do
  X := X - 1
end
{{ X = 0 }}
```

```
[#requires …]
[#ensures …]
fn target(x: T₁…) -> T
```

```
VERIFIER_assume(true);
while (X ≠ 0) {
  X = X - 1;
}
VERIFIER_assert(X = 0);
```

Verification of Program with a Specification

{Pre-condition} Program {Post-condition}

$$\{P\}\ c\ \{Q\}$$

Program c: a program state transformer
Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

P $\quad x$: Int

c

```
int abs(int x) {
  int y;
  if (x >= 0) // B1
    y = x; // B2
  else
    y = -x; // B3
  return y; // B4
}
```

Q $\quad (y = -x \lor y = x) \land y \geq 0$

Program c: a program state transformer
Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
```

$\{x: \text{Int}\}$

```
  int y;

  if (x >= 0) // B1

    y = x; // B2

  else

    y = -x; // B3

  return y; // B4
}
```

Goal: $(y = -x \lor y = x) \land y \geq 0$

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
```

$\{x\colon \mathrm{Int}\}$

```
  int y;
```

$\{x\colon \mathrm{Int}, y\colon \mathrm{Int}\}$

```
  if (x >= 0) // B1

    y = x; // B2

  else

    y = -x; // B3

  return y; // B4
}
```

Goal: $(y = -x \lor y = x) \land y \geq 0$

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2

  else

    y = -x; // B3

  return y; // B4
}
```

Goal: $(y = -x \lor y = x) \land y \geq 0$

Program c: a program state transformer
Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
```

$\{x: \mathrm{Int}\}$

```
  int y;
```

$\{x: \mathrm{Int}, y: \mathrm{Int}\}$

```
  if (x >= 0) // B1
```

$\{x: \mathrm{Int}, y: \mathrm{Int}, x \geq 0\}$

```
    y = x; // B2
```

$\{x: \mathrm{Int}, y: \mathrm{Int}, x \geq 0, x = y\}$

```
  else

    y = -x; // B3

  return y; // B4
}
```

Goal: $(y = -x \vee y = x) \wedge y \geq 0$

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2
    {x: Int, y: Int, x ≥ 0, x = y}
  else
    {x: Int, y: Int, x < 0}
    y = -x; // B3

  return y; // B4
}
```

Goal: $(y = -x \vee y = x) \wedge y \geq 0$

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2
    {x: Int, y: Int, x ≥ 0, x = y}
  else
    {x: Int, y: Int, x < 0}
    y = -x; // B3
    {x: Int, y: Int, x < 0, y = -x}
  return y; // B4
}
```

Goal: $(y = -x \vee y = x) \wedge y \geq 0$

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
```

$\{x: \mathrm{Int}\}$

```
int y;
```

$\{x: \mathrm{Int}, y: \mathrm{Int}\}$

```
if (x >= 0) // B1
```

$\{x: \mathrm{Int}, y: \mathrm{Int}, x \geq 0\}$

```
y = x; // B2
```

$\{x: \mathrm{Int}, y: \mathrm{Int}, x \geq 0, x = y\}$

```
else
```

$\{x: \mathrm{Int}, y: \mathrm{Int}, x < 0\}$

```
y = -x; // B3
```

$\{x: \mathrm{Int}, y: \mathrm{Int}, x < 0, y = -x\}$

$\{x: \mathrm{Int}, y: \mathrm{Int}, (x < 0 \land y = -x) \lor (x \geq 0 \land x = y)\}$

```
return y; // B4
}
```

Goal: $(y = -x \lor y = x) \land y \geq 0$

Program c: a program state transformer
Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2
    {x: Int, y: Int, x ≥ 0, x = y}
  else
    {x: Int, y: Int, x < 0}
    y = -x; // B3
    {x: Int, y: Int, x < 0, y = -x}
  {x: Int, y: Int, (x < 0 ∧ y = -x) ∨ (x ≥ 0 ∧ x = y)}
  return y; // B4
}
```

Goal: $(y = -x \lor y = x) \land y \geq 0$

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2
    {x: Int, y: Int, x ≥ 0, x = y}
  else
    {x: Int, y: Int, x < 0}
    y = -x; // B3
    {x: Int, y: Int, x < 0, y = -x}

  return y; // B4
}
```

$\{x\text{: Int}, y\text{: Int}, (x < 0 \land y = -x) \lor (x \geq 0 \land x = y)\}$

$$\Downarrow$$

Goal: $(y = -x \lor y = x) \land y \geq 0$

Program c: a program state transformer
Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2
    {x: Int, y: Int, x ≥ 0, x = y}
  else
    {x: Int, y: Int, x < 0}
    y = -x; // B3
    {x: Int, y: Int, x < 0, y = -x}

  return y; // B4
}
```

$\{x\colon \mathrm{Int}, y\colon \mathrm{Int}, (x < 0 \land y = -x) \lor (x \geq 0 \land x = y)\}$

$\Downarrow$

Goal: $(y = -x \lor y = x) \land y \geq 0$

?

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2
    {x: Int, y: Int, x ≥ 0, x = y}
  else
    {x: Int, y: Int, x < 0}
    y = -x; // B3
    {x: Int, y: Int, x < 0, y = -x}

  return y; // B4
}
```

$\{x\text{: Int}, y\text{: Int}, (x < 0 \land y = -x) \lor (x \geq 0 \land x = y)\}$

$$\Downarrow$$

Goal: $(y = -x \lor y = x) \land y \geq 0$

Z3 Theorem Prover!

Can you write a python program with Z3 to check the satisfiability of the following

$\{x{:}\mathrm{Int}, y{:}\mathrm{Int}, (x<0 \wedge y=-x) \vee (x \geq 0 \wedge x=y)\}$ => Goal: $(y=-x \vee y=x) \wedge y \geq 0$

Thought for 16s

Here's a tiny Z3Py script that checks **validity** of your implication by asking Z3 to find a counterexample to `pre ∧ ¬goal`. If that is **unsat**, then the implication holds for all integers `x, y`.

python Copy code

```python
# pip install z3-solver  (if not already installed)
from z3 import *

# Vars
x, y = Ints('x y')

# Specification
pre  = Or(And(x < 0, y == -x),
          And(x >= 0, x == y))

goal = And(Or(y == -x, y == x),
           y >= 0)

# Check validity by searching for a counterexample to pre → goal
s = Solver()
s.add(pre, Not(goal))
res = s.check()
```

Program c: a program state transformer

Pre- and Post-condition P, Q: a Boolean function over program states

$$\forall x, P(x) \Rightarrow Q(c(x))$$

```
int abs(int x) {
  {x: Int}
  int y;
  {x: Int, y: Int}
  if (x >= 0) // B1
    {x: Int, y: Int, x ≥ 0}
    y = x; // B2
    {x: Int, y: Int, x ≥ 0, x = y}
  else
    {x: Int, y: Int, x < 0}
    y = -x; // B3
    {x: Int, y: Int, x < 0, y = −x}

  return y; // B4
}
```

$\{x\colon \mathrm{Int}, y\colon \mathrm{Int}, (x < 0 \wedge y = -x) \vee (x \geq 0 \wedge x = y)\}$

$$\Downarrow$$

Goal: $(y = -x \vee y = x) \wedge y \geq 0$

Z3 Theorem Prover!

✅

Software Foundations: Hoare Logic (Coq)

```
{{ True }} ->>
{{ m = m }}
  X := m
                          {{ X = m }} ->>
                          {{ X = m ∧ p = p }};
  Z := p;
                          {{ X = m ∧ Z = p }} ->>
                          {{ Z - X = p - m }}
  while X ≠ 0 do
                          {{ Z - X = p - m ∧ X ≠ 0 }} ->>
                          {{ (Z - 1) - (X - 1) = p - m }}
    Z := Z - 1
                          {{ Z - (X - 1) = p - m }};
    X := X - 1
                          {{ Z - X = p - m }}
  end
{{ Z - X = p - m ∧ ¬ (X ≠ 0) }} ->>
{{ Z = p - m }}
```

```
A {l.len > 0 | self: List(l)}
  match self { // Destr.List
    List::Nil => {
    B {l.len > 0 ∧ l = {δ: Nil, len: 0} | emp}
    D {false | emp}
      unreachable!()       // Unreachable
    C {result: T}}
    List::Cons { elem, next } => {
    E {l = ... | elem: T ∗ next: Box}
      drop!(next);         // Drop
     {l = ... | elem: T}
      let result = elem;  // Rename
    F {l = ... | result: T}
    C {result: T}}
  {result: T}
```

Leveraging Rust Types for Program Synthesis (Fiala et. al., 2023)

Z3 Theorem Prover!

Satisfiability Modulo Theories (SMT) Solver

Others: CVC3, CVC4, CVC5, Beaver, UCLID, veriT, ...

$$\{P\}\ c\ \{Q\}$$

Proof-Theoretic Semantics

Z3 Theorem Prover!

Satisfiability Modulo Theories (SMT) Solver

Others: CVC3, CVC4, CVC5, Beaver, UCLID, veriT, ...

Specification language

Syntax + Semantics

- First-order logic operators:
  - $\forall, \exists, =, \neq, \wedge, \vee, \neg, \Rightarrow, \ldots$
- Base types, commonly used types, and their predicates
  - Int, Bool, Set[T], List[T], ...
  - $+, -, \times, \div, ::, \cup, \cap, \in, [.], \&\&, ||, !, len, \ldots$
- Logic systems to specify program behaviors
  - Memory (Separation Logic): $\{.\}, \mapsto, *, \ldots$
  - Temporal Behavior (Temporal Logic): $\mathrm{Globally}, \mathrm{Next}, \mathrm{Finally}, \ldots$
  - Mathematical Objects: Permutation, ...

$$\{P\}\ c\ \{Q\}$$

Hoare Triple

Functional Specification Guided Synthesis

Structural Specification

Target Language

Syntax & Semantics

Types

Functional Specification Guided Synthesis

Structural Specification
Target Language
Syntax & Semantics
Types

→

Synthesizer
Bottom-up Enumerator
Top-Down Enumerator
Deductive Synthesis
Large Language Model

Functional Specification Guided Synthesis

Program

Structural Specification
Target Language
Syntax & Semantics
Types

Synthesizer
Bottom-up Enumerator
Top-Down Enumerator
Deductive Synthesis
Large Language Model

Verifier
Z3
CVC5
UCLID

Functional Specification Guided Synthesis

Program

Structural Specification
Target Language
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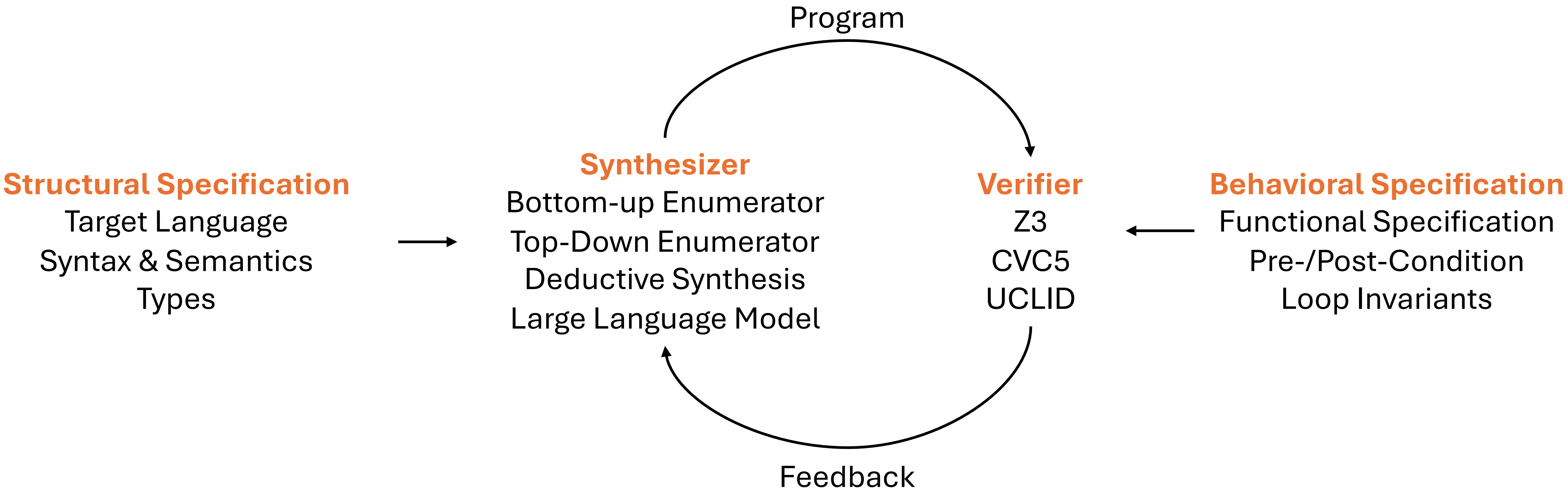
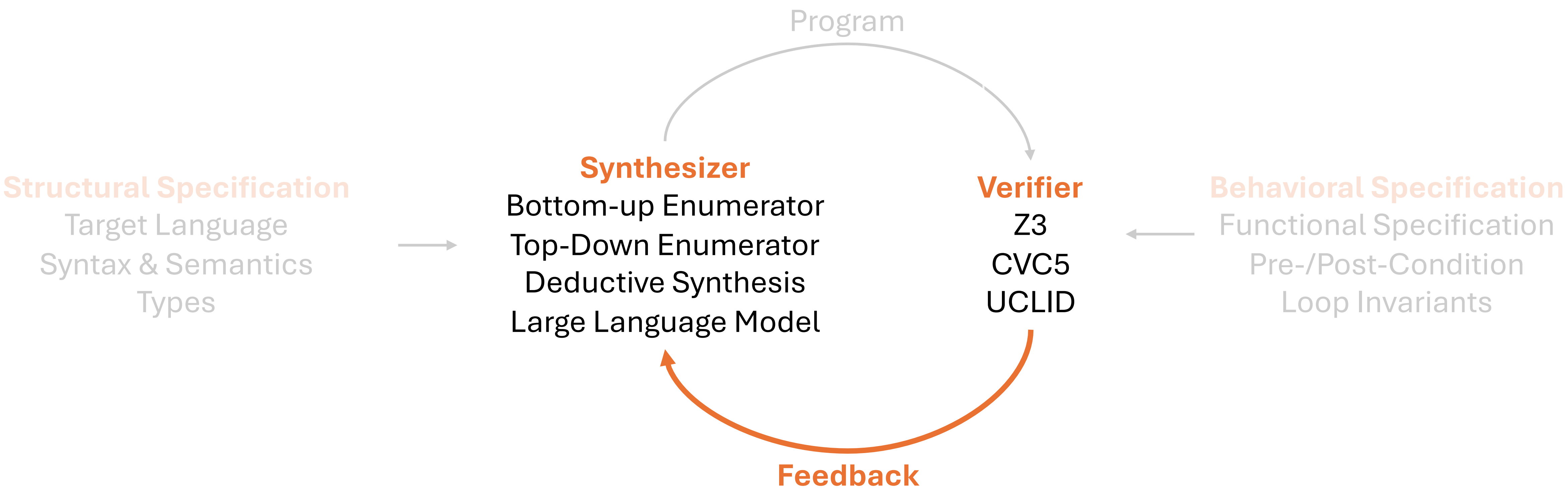
Verifier
Z3
CVC5
UCLID

Behavioral Specification
Functional Specification
Pre-/Post-Condition
Loop Invariants

Functional Specification Guided Synthesis



Functional Specification Guided Synthesis



Verifier’s Feedback

$\{P\}\ c\ \{Q\}$

Verifier’s Feedback

$\{P\}\ c\ \{Q\} \quad \longrightarrow$

Z3 Theorem Prover!

Verifier's Feedback

$\{P\}\ c\ \{Q\} \longrightarrow$

Z3 Theorem Prover!

SAT

UNSAT

Timeout

Verifier’s Feedback

$\{P\}\ c\ \{Q\} \longrightarrow$ **Z3 Theorem Prover!**

- $\nearrow$ SAT $\longrightarrow$ Example
- $\rightarrow$ UNSAT
- $\searrow$ Timeout

Verifier's Feedback

$\forall x, P(x) \Rightarrow Q(c(x)) \longrightarrow$ **Z3 Theorem Prover!**

- SAT $\longrightarrow$ Example
- UNSAT
- Timeout

Verifier's Feedback

$\forall x, P(x) \Rightarrow Q(c(x)) \longrightarrow$ Z3 Theorem Prover!

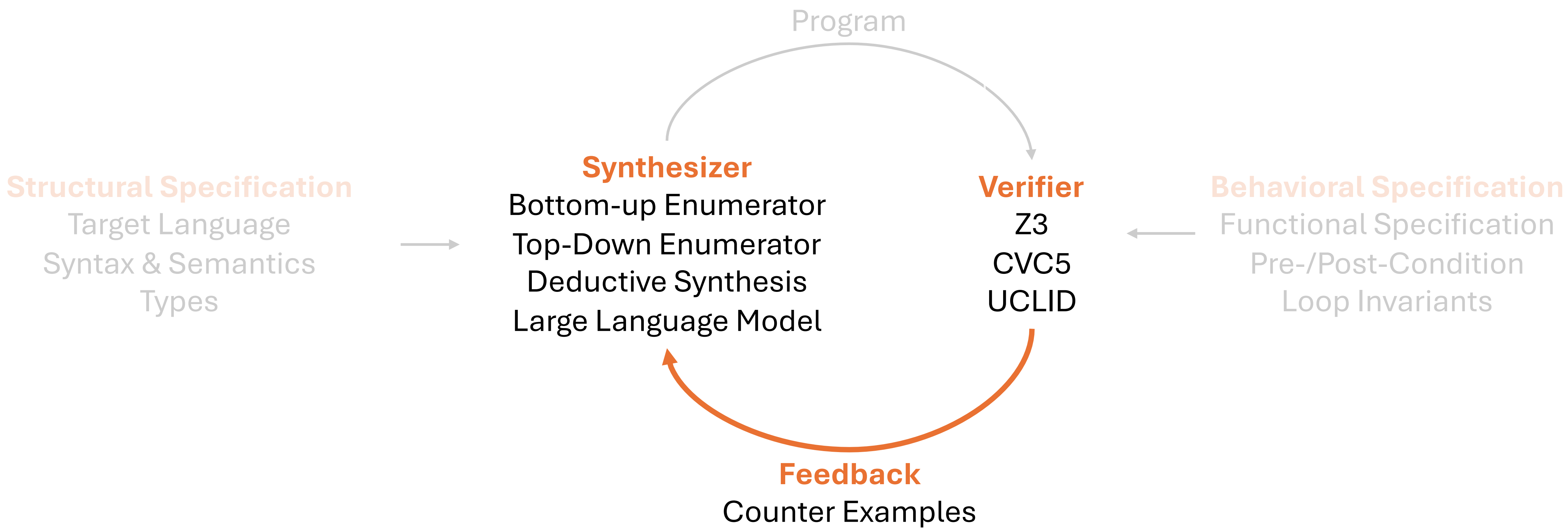
- SAT $\longrightarrow$ Example
- UNSAT
- Timeout

$\exists x, P(x) \land \neg Q(c(x)) \longrightarrow$ Z3 Theorem Prover!

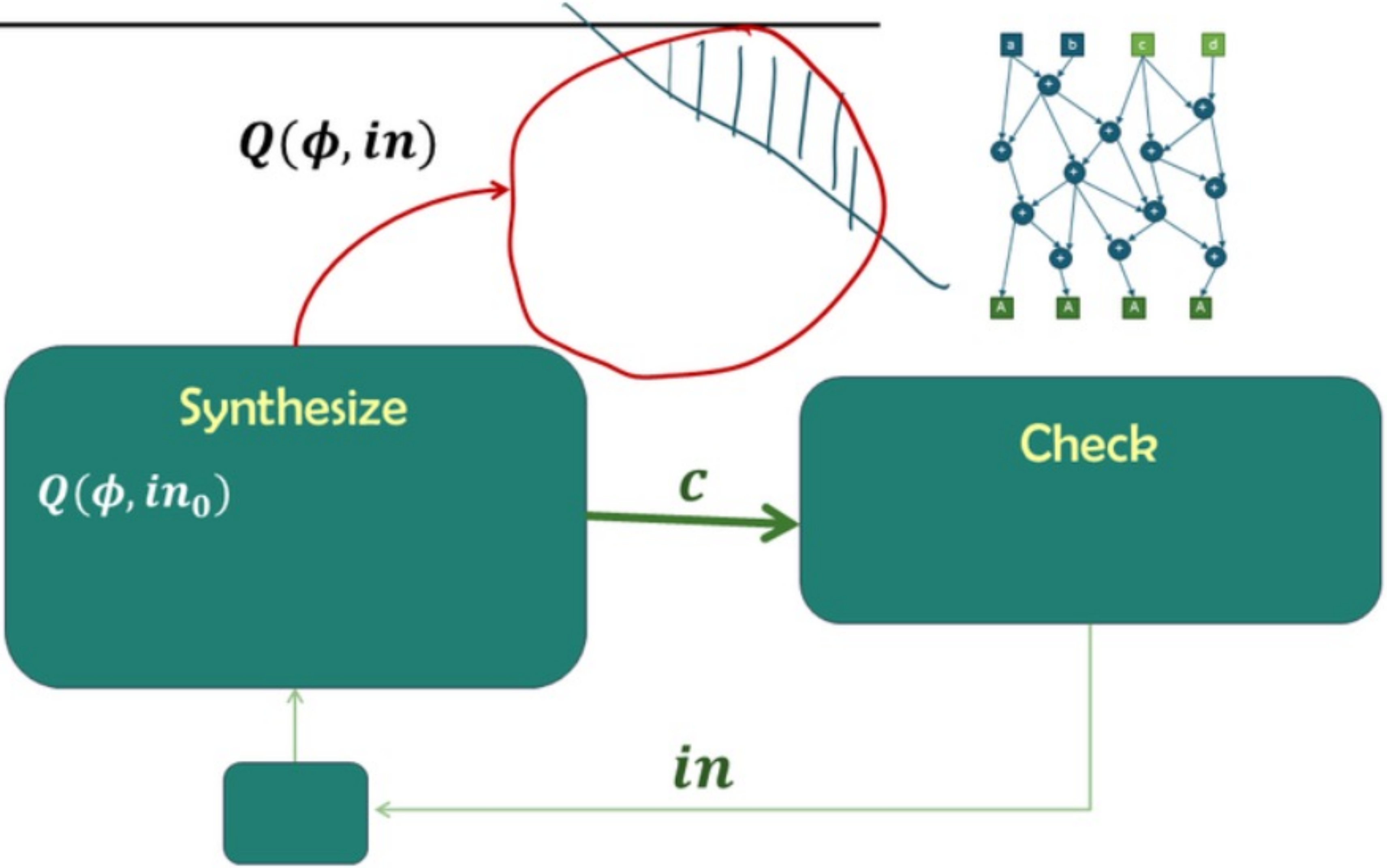
- SAT $\longrightarrow$ **Counter Example**
- UNSAT
- Timeout

```python
x, y = Ints('x y')

# Original pre and goal
pre  = Or(And(x < 0, y == -x),
          And(x >= 0, x == y))

goal = And(Or(y == -x, y == x),
           y >= 0)

# 1) Try to find a counterexample to the original implication
s = Solver()
s.add(pre, Not(goal))
r = s.check()
print("Counterexample to original (pre → goal)?", r)
if r == sat:
    m = s.model()
    print("Counterexample model:", "x =", m[x], ", y =", m[y])
else:
    print("No counterexample; implication is VALID.")
```

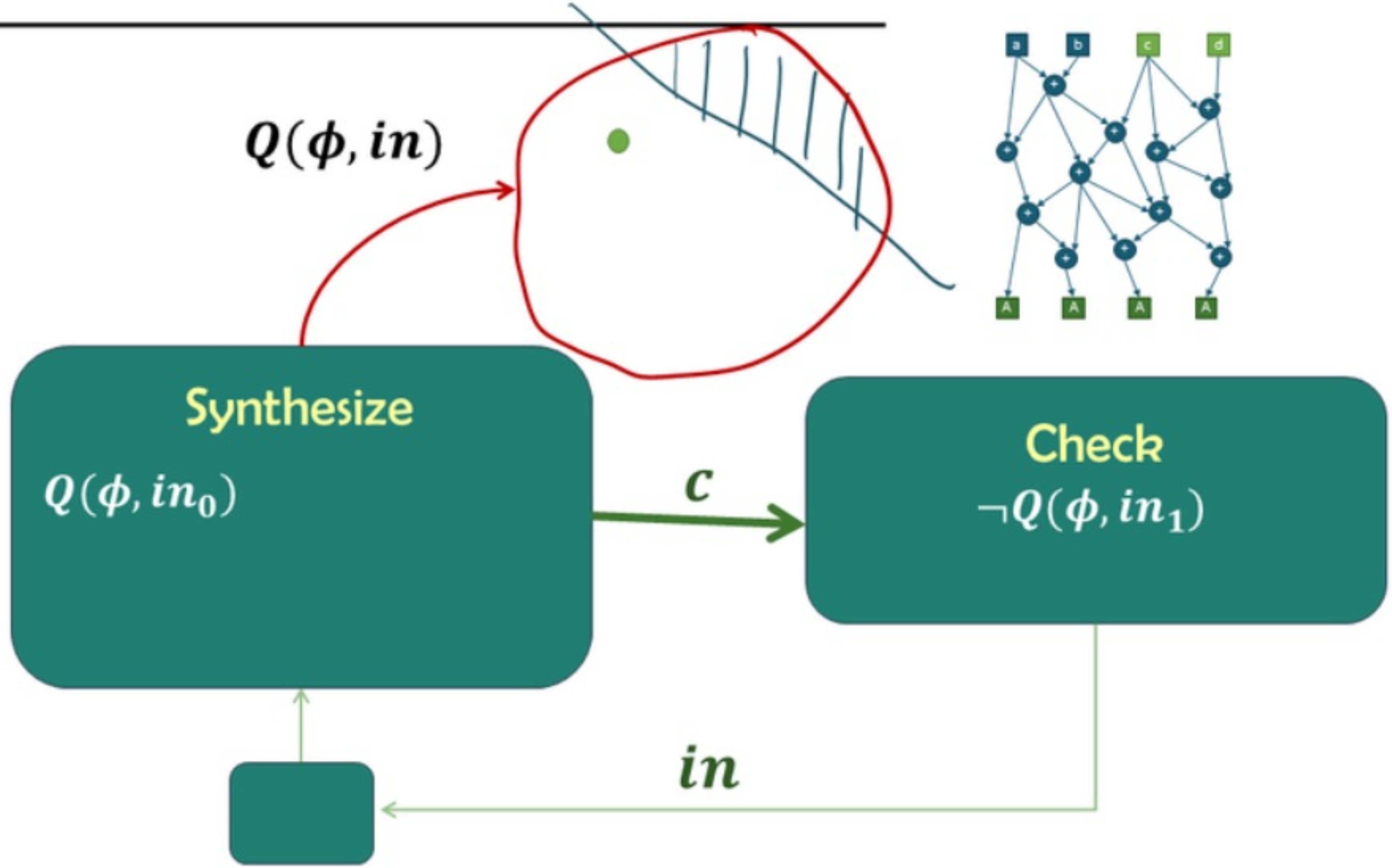
Functional Specification Guided Synthesis



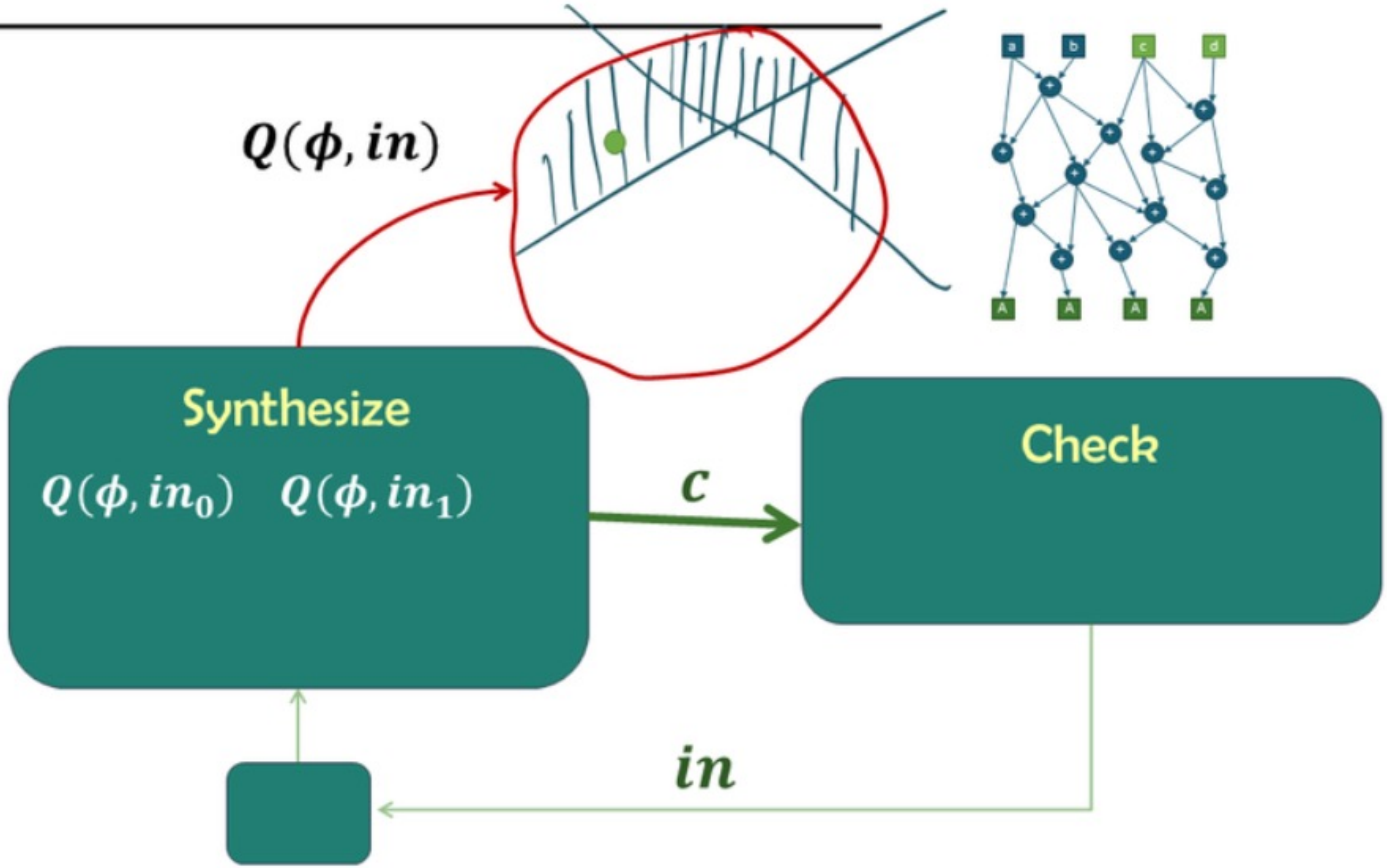
Constraint-based CEGIS



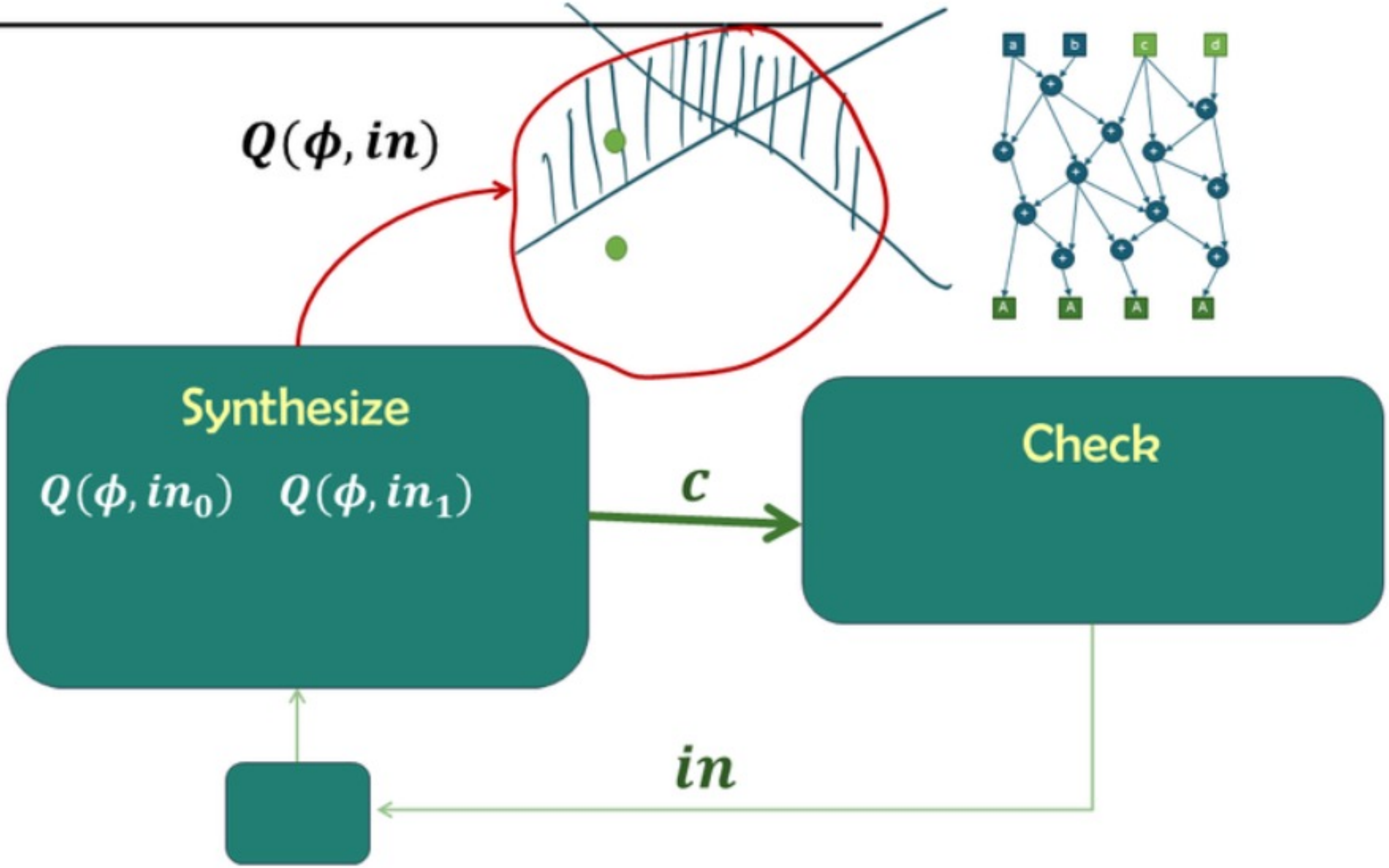
Constraint-based CEGIS



Constraint-based CEGIS



Constraint-based CEGIS



Notes

- There is NO FREE LUNCH!
  - Where do you get the functional specifications?
    - Where do you get the language for functional specifications? Is it expressive enough to model program behaviors?
    - Is your functional specification complete?
  - How do you deal with Loops?
    - Loops need “Loop Invariants” for an automated theorem prover to function
    - How do you get the loop invariants?
  - Z3: How long does it take for Z3 to verify {P} c {Q}?
    - Often time, Z3 cannot deal with extremely large programs, and will timeout
    - Who is doing the simplification?
  - Verification <> Synthesis interplay overhead and deficiency?

Notes

- “Multi-Modal” Program Synthesis

Structural Specification
Target Language
Syntax & Semantics
Partial Program; Templates
...

Behavioral Specification
Examples
Counter Examples
Functional Specifications
Refinement Types
Natural Language
...

Synthesizer
Bottom-up Enumerator
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Notes

- “Multi-Modal” Program Synthesis

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Notes

- Refinement Type as Functional Specification:

```
def abs(x: int) -> int:
  …

def abs(x: int) -> {v: int | v >= 0}:
  …
```

Week 2

- Assignment 1
  - https://github.com/machine-programming/assignment-1
  - Autograder Up on GradeScope (Finally!)
  - There is OOM issue on GradeScope that you need to manage!
  - Due Date Sep 16 (Extended for 5 days from Sep 11)
- Logistics
  - There should be an increase in class capacity (25-30)
  - Everyone who has been following should be able to get in
  - Send me an email if you are newly enrolled